Application of Formal Concept Analysis in Security Engineering

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ABSTRACT

The research paper deals with the application of formal concept analysis to security engineering for the efficient selection of an appropriate component. The research part is devoted to the application of the Concept Explorer software, based on formal conceptual analysis, on selected elements of the circuit protection which are available on the market. The last section contains the results and the data commentary.

Keywords: formal concept analysis, conceptual clusters, concept lattice, attribute implications, software Concept Explorer, security engineering.

1 INTRODUCTION

The formal concept analysis (FCA) (Ganter & Wille, 1999) has become a popular method of data analysis in engineering because of visualization of data dependency in graphical models. The models provide a comprehensive overview of objects and their attributes. Therefore, it has been applied to many problems such as:

- Software engineering (Hesse & Tilley, 2005)
- Data mining and knowledge discovery (Eklund, 2004)
- Information Retrieval (Ignatov, 2014)
- Hierarchical organization of web search results (Carpineto & Romano, 2005)
- Analysis and visualization of data related to terrorist activities (Farley, 2003) [1,2]

In recent years, completely automated tools and techniques have been used in data analysis. However, these methods assume an often absent clear definition of available terms in the underlying data. When the data are not exactly defined and the survey aims are vague, the data visualization and the visual analysis can be very useful.

The purpose of this paper is to present FCA as an explorative data analysis method showing non-trivial information about input data that may not be obvious at first glance.

2 THE FCA METHOD

We used the formal concept analysis (FCA) method which was invented in 1982 by Rudolf Wille as a mathematical theory. It is based on lattice theory as a branch of applied mathematics and it provide the analysis of table data from which two kinds of outputs are produced. The first output is a concept lattice and the second one is a collection of attribute implications. These outputs are the goals of FCA.

This methods section provides information about mathematical foundations and the software tool standing behind FCA that helps readers understand the data processing.

2.1 Mathematical Foundations

FCA starts with producing a cross-table known as a formal context. It is a triplet $\langle X, Y, I \rangle$ representing the input data table where X is a set of objects, Y is a set of bivalent logical attributes and I is a binary relation between X and Y. [3]

For instance, Table 1 depicts the triplet $\langle X, Y, I \rangle$ given by elements of X = $\{x_1, x_2, x_3, x_4, x_5\}$ and elements of Y = $\{y_1, y_2, y_3, y_4, y_5\}$. The relationship I is put by the logical bivalent value 1, which indicates that object x has attribute y, and 0 that means object x does not have attribute y.

Table 1. Formal Context [4]

I	y_1	y_2	y_3	y_4
x_1	1	1	1	1
x_2	1	0	1	1
x_3	0	1	1	1
x_4	0	1	1	1
x_5	1	0	0	0

In every formal context, clusters called formal concepts that share all attributes occur. The formal concept is a pair $\langle A, B \rangle$ where A includes just objects sharing all attributes from B, and B includes just attributes shared by all objects from A.

Considering the formal context ⟨X, Y, I⟩ in Table 1, there are following formal concepts:

$\langle A_1, B_1 \rangle = \langle \{x_1, x_2, x_3, x_4\}, \{y_3, y_4\} \rangle$

$\langle A_2, B_2 \rangle = \langle \{x_1, x_3, x_4\}, \{y_2 y_3, y_4\} \rangle$

$\langle A_3, B_3 \rangle = \langle \{x_1, x_2\}, \{y_1, y_3, y_4\} \rangle$

$\langle A_4, B_4 \rangle = \langle \{x_1, x_2, x_5\}, \{y_1\} \rangle$ [1]

Table 2. Formal Concepts included in formal context [4]

(A_1,B_1)					(A_2,B_2)				
I	y_1	y_2	y_3	y_4	I	y_1	y_2	y_3	y_4
x_1	1	1	1	1	x_1	1	1	1	1
x_2	1	0	1	1	x_2	1	0	1	1
x_3	0	1	1	1	x_3	0	1	1	1
x_4	0	1	1	1	x_4	0	1	1	1
x_5	1	0	0	0	x_5	1	0	0	0
(A_3,B_3)					(A_4,B_4)				
I	y_1	y_2	y_3	y_4	I	y_1	y_2	y_3	y_4
x_1	1	1	1	1	x_1	1	1	1	1
x_2	1	0	1	1	x_2	1	0	1	1
x_3	0	1	1	1	x_3	0	1	1	1
x_4	0	1	1	1	x_4	0	1	1	1
x_5	1	0	0	0	x_5	1	0	0	0

Formal concepts are ordered by subconcept-superconcept relation defined by an operator ≤ as follows:

$\langle A_2, B_2\rangle \leq \langle A_1, B_1\rangle$ if $A_2 \subseteq A_1$ (if $B_1 \subseteq B_2$)

Based on this rule, the following ordering can be determined: $\langle A_3, B_3\rangle \leq \langle A_1, B_1\rangle$, $\langle A_3, B_3\rangle \leq \langle A_2, B_2\rangle$, $\langle A_3, B_3\rangle \leq \langle A_4, B_4\rangle$, $\langle A_1, B_1\rangle \parallel \langle A_4, B_4\rangle$, $\langle A_2, B_2\rangle \parallel \langle A_4, B_4\rangle$. The operator ∥ means that the formal concepts are incomparable. [1]

In conclusion, the collection of all formal concepts covered in formal context is called a concept lattice.

2.2 Software Concept Explorer

Concept Explorer is the available FCA software for data processing which allows:

- Context editing
- Creation of concept lattice embedded within the context
- To find implications or association rules contained in the context
- To perform an attribute survey

Before conducting the analysis, it was necessary to have the data in a form of simple cross-table, as shown in Figure 1, where objects are represented by table rows and attributes are represented by columns. If the attribute belongs to the given object, the box in the table is marked with a cross (corresponds to bivalent value 1), and if it is the opposite, the box remains empty (corresponds to bivalent value 0). Then it is possible to create the conceptual lattice and the attribute implications.

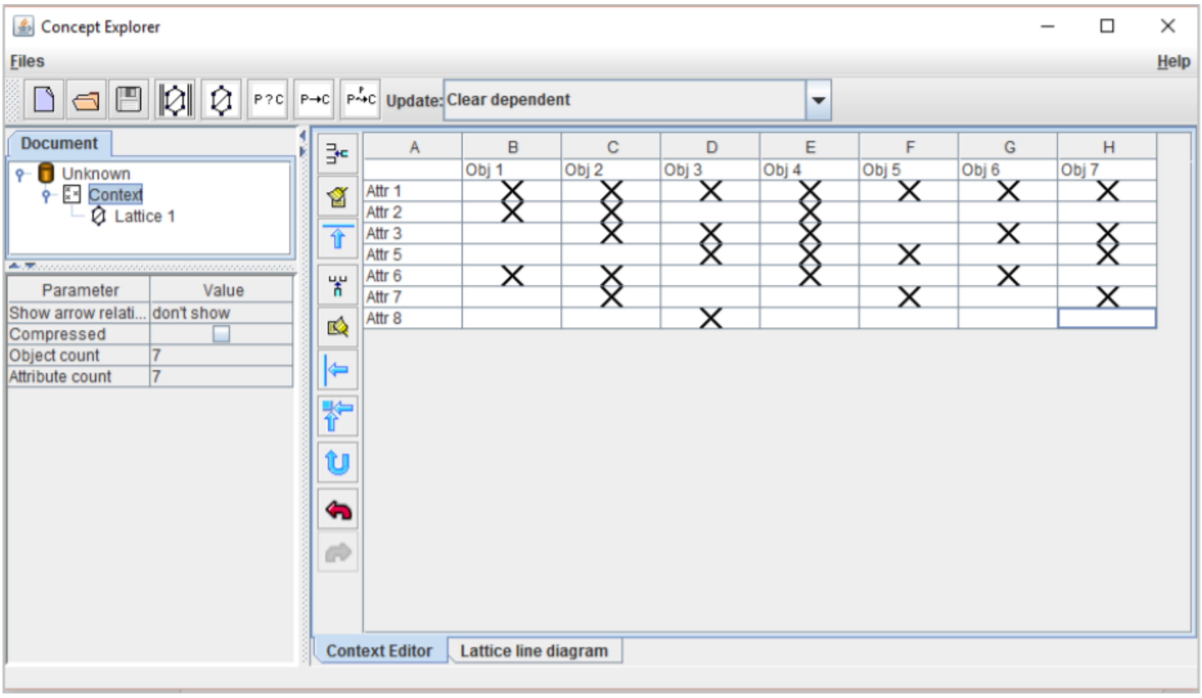

Figure 1. Cross-table in the ConExp [4]

3 RESULTS

This section shows the results of FCA applied to security engineering for the efficient selection of an appropriate component type. Two types of output are produced by Concept Explorer from the input data:

The first is a concept lattice representing a set of hierarchically arranged formal concepts in the data given. The conceptual lattice in Figure 1 depicts the relationship between objects (Stratobel Security Burglary glasses marked as SSRB) and their attributes (resistance class, total thickness, weight, acoustic isolation, and dimension).

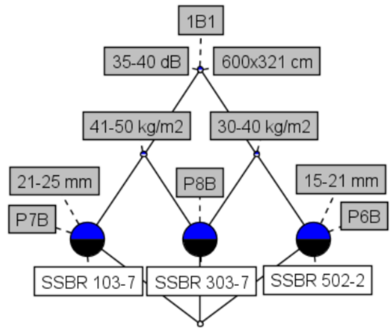

Figure 1. Concept Lattice [4]

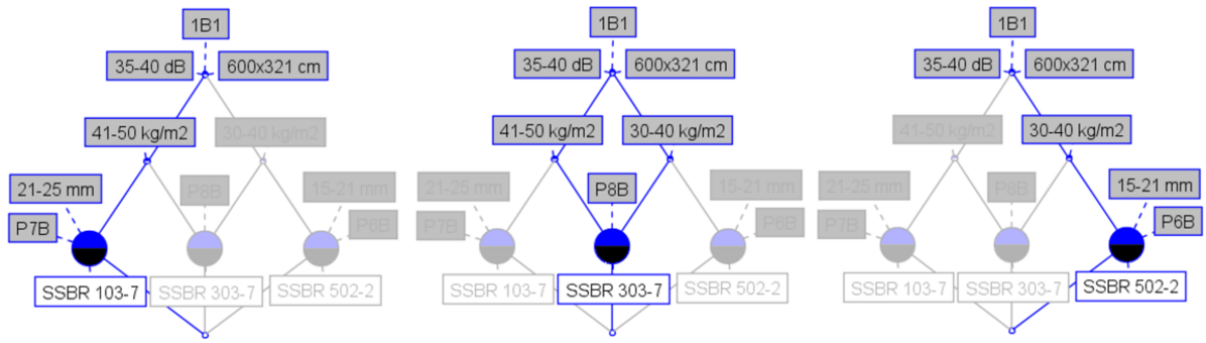

Figure 2. Navigating in concept lattice [4]

In Figure 1, objects (white rectangles) and attributes (grey rectangles) are connected as defined in the context table. A blue semicircle marks a connection with an attribute, a black semicircle marks a connection with the object.

The second output are attribute implications describing specific dependencies valid in the data.

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1 < 3 > { } ==> 1B1 35-40 dB 600x321 cm;
2 < 1 > 1B1 30-40 kg/m2 41-50 kg/m2 35-40 dB 600x321 cm ==> P8B;
3 < 1 > 1B1 21-25 mm 35-40 dB 600x321 cm ==> P7B 41-50 kg/m2;
4 < 1 > 1B1 15-21 mm 35-40 dB 600x321 cm ==> P6B 30-40 kg/m2;
5 < 1 > 1B1 P8B 35-40 dB 600x321 cm ==> 30-40 kg/m2 41-50 kg/m2;
6 < 1 > 1B1 P7B 35-40 dB 600x321 cm ==> 21-25 mm 41-50 kg/m2;
7 < 0 > 1B1 P7B P8B 21-25 mm 30-40 kg/m2 41-50 kg/m2 35-40 dB 600x321 cm ==> P6B 15-21 mm;
8 < 1 > 1B1 P6B 35-40 dB 600x321 cm ==> 15-21 mm 30-40 kg/m2;
9 < 0 > 1B1 P6B P8B 15-21 mm 30-40 kg/m2 41-50 kg/m2 35-40 dB 600x321 cm ==> P7B 21-25 mm;
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Figure 3. Attribute implications [4]

4 DISCUSSION

The research suggests that the formal concept analysis (FCA) can be a relatively simple way to explore extensive data.

In this paper, the FCA was applied to security engineering with the intent to select a higher level of resistance of the circuit element. To achieve this aim, the object-attribute data was converted to graphical models in which we can display the appropriate dependencies related to the required object or attribute through the navigating in the context union.

In agreement with other studies, e.g. Ganter & Wille (1999), the results lead to complex visualization of data dependencies that might not have been clear at the first moment.

It should be noted that the FCA works only with sets of classic set theory which is its limitation.

Therefore, future research should be focused on special extending of FCA by fuzzy theory describing fuzzy attributes that are more common in the real world.

5 ACKNOWLEDGMENT

This work was supported by the Internal Grant Agency of Tomas Bata University under the project No. IGA/CebiaTech/2020/003.

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