Algebraic Procedures in Integral Calculus and Ordinary Differential Equations

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Abstract
In a preparation of future teachers at faculties of education, courses based on the mathematical theory can have a different form, which depends on a particular study program. An interesting situation can occur in case of education of foreign students. These students can have different experiences in a mathematical field of study. Although a mathematical education of foreign students can include individual approaches to students, an improving of educational schemes is a necessary trend. Especially in case of courses based on advanced parts of mathematics, modified approaches to education are required. In this paper, a proposal of using algebraic procedures is presented in favor of course of mathematical analysis. Finding roots and solving set of linear equations can be implemented using software Octave and then teacher can focus on an explanation of more difficult parts of mathematical analysis e.g. in integral calculus or in ordinary differential equations.

Keywords
Mathematical Education, Algebra, Integral Calculus, Ordinary Differential Equations, Octave

Introduction

Modifications of educational approaches can be advantageous in case of international exchange programs at universities. Modified approaches can be applied in particular courses focused on education of foreign students. A structure of these courses should be related to their study programs, which can be various. For purposes of increasing quality of education, proposals focused on educational strategies have been improved yet. As a special case, a professional preparation of future teachers (foreign students) can be considered. In this paper, a proposal of a simplification of particular procedures is presented in educational strategies, which can be applied in advanced parts of mathematics at faculties of education. However, this proposal can be implemented at technical universities.

In advanced parts of mathematics, students from faculties of education often require only an explanation of essential methods. This situation can cause an absence of advanced principles, which can be seen e.g. in mathematical analysis. Usually, their degree programs are consisted of an introduction to mathematics. In following study programs, students attend a practice and didactical courses. Some universities can have similar aims in curriculum of mathematics as at technical universities. Concretely, a higher level of student's knowledge has influence on a final form of education of attending students. For purposes of integration of knowledge of each student in a course, proposals with improving aspects can be advantageous.

As more difficult topics in advanced parts of mathematics, integral calculus [1] and ordinary differential equations [2] with algebraic procedures [3] can be considered. In limits [1] or derivatives [1], there are not so additional algebraic operations in particular procedures. In integral calculus and in ordinary differential equations, algebraic procedures are frequently appeared. For purposes of a simplification, these algebraic procedures can be solved using modular approaches.

Modifications of didactical concepts focused on education of foreign students have not been so widely described yet. In this context, a particular proposal is further presented in this contribution. A presented strategy causes a modular approach to solving of essential algebraic procedures using the free-available software Octave [4]. Advanced particular problems in integral calculus or in ordinary differential equations can be simplified using this computational support. More difficult topics of mathematics can be understood by students. Therefore, proposals based on utilization of software Octave [4] can be advantageous in context of mathematical education at universities of various types.

Methods in Mathematical Analysis with Proposal of Processing Algebraic Procedures

In this paper, following algebraic operations [3] are considered as carrier: finding roots and solving set of linear equations. In software Octave [4], these algebraic procedures can be processed using command Roots (1), respectively (2) in case of solving set of linear equations using command Inv (for inversion of matrix $\boldsymbol{A}$). Octave is appropriate also for other parts of mathematical education even in control system theory at technical faculties [5]. Including commands of Octave into mathematical expressions is further denoted using a triple equality sign in (2) and software commands are written by Courier New font in this paper.

$$\left.\begin{aligned} &\texttt{Roots}([a_n\ a_{n-1}\cdots a_1\ a_0]);\\ &a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0 \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} &\boldsymbol{k} = \left[k_1 \cdots k_r\right]^T \equiv \texttt{inv}\,(\boldsymbol{A}) * \boldsymbol{b};\\ &\boldsymbol{A}\boldsymbol{k} = \boldsymbol{b};\ \boldsymbol{A} \in \mathcal{R}^{r,r};\ \boldsymbol{k} \in \mathcal{R}^{r,1}\ \boldsymbol{b} \in \mathcal{R}^{r,1} \end{aligned}\right\} \tag{2}$$

In integral calculus [1], problems of partial fractions can be based on finding roots [3]. In case (3) with polynomial of the second order, routine operation (1) can be advantageous in favor of an explanation of the method of partial fractions. In mathematical examples, complicated fractions can be further divided in form with polynomials of the first orders with roots (4), as can be seen in (5), in favor of a better understanding of main procedure (5)-(10). In step (6), method of undetermined coefficients [1] can be seen with application of Octave command for solving set of linear equations (7) using rule (2). After using substitution method (9), suitable form (10) could be solved more efficient by described rules using Octave in favor of understanding integral rules.

$$\int \frac{b}{a_2 x^2 + a_1 x + a_0}\,dx;\ \boldsymbol{a} = \left[a_2\ a_1\ a_0\right]^T \in \mathcal{R}^{3,1} \tag{3}$$

$$\left.\begin{aligned} &\boldsymbol{m} = [m_1\ m_2]^T \equiv \texttt{Roots}([a_2\ a_1\ a_0]);\ \boldsymbol{m} \in \mathcal{R}^{2,1}\\ &a_2 x^2 + a_1 x + a_0 = a_2(x - m_1)(x - m_2) \end{aligned}\right\} \tag{4}$$

$$\int \frac{t_1}{a_2(x - m_1)}\,dx + \int \frac{t_2}{x - m_2}\,dx;\ \boldsymbol{t} = [t_1\ t_2]^T;\ \boldsymbol{t} \in \mathcal{R}^{2,1} \tag{5}$$

$$\left.\begin{aligned} &\frac{b}{a_2 x^2 + a_1 x + a_0} = \frac{t_1}{a_2(x - m_1)} + \frac{t_2}{x - m_2}\\ &b = t_1(x - m_2) + a_2 t_2 (x - m_1)\\ &\underline{0}x + \underline{\underline{b}} = x\underline{(t_1 + a_2 t_2)} - \underline{\underline{(t_1 m_2 + a_2 t_2 m_1)}} \end{aligned}\right\} \tag{6}$$

$$\left.\begin{array}{c} \boldsymbol{t} = [t_1\ t_2]^T \equiv \texttt{inv}\,(\boldsymbol{D}) * \boldsymbol{\delta}; \\ \boldsymbol{D}\boldsymbol{t} = \boldsymbol{\delta}; \boldsymbol{D} \in \mathcal{R}^{2,2}; \boldsymbol{t} \in \mathcal{R}^{2,1}\ \boldsymbol{\delta} \in \mathcal{R}^{2,1} \\ \boldsymbol{D} = \begin{bmatrix} 1 & a_2 \\ -m_2 & -a_2 m_1 \end{bmatrix}; \boldsymbol{\delta} = \begin{bmatrix} 0 \\ b \end{bmatrix} \\ \boldsymbol{t} = \boldsymbol{D}^{-1}\boldsymbol{\delta} \end{array}\right\} \tag{7}$$

$$\int \frac{t_1}{a_2(x-m_1)}dx + \int \frac{t_2}{x-m_2}dx = \frac{t_1}{a_2}\int \frac{1}{x-m_1}dx + t_2 \int \frac{1}{x-m_2}dx \tag{8}$$

$$\left.\begin{array}{c} \frac{t_1}{a_2}\int \frac{1}{x-m_1}dx + t_2 \int \frac{1}{x-m_2}dx \\ \left| \begin{array}{c} x - m_1 = \mu_1 \\ dx = d\mu_1 \end{array} \right|; \left| \begin{array}{c} x - m_2 = \mu_2 \\ dx = d\mu_2 \end{array} \right| \\ \frac{t_1}{a_2}\int \frac{1}{\mu_1}d\mu_1 + t_2 \int \frac{1}{\mu_2}d\mu_2 \end{array}\right\} \tag{9}$$

$$\frac{t_1}{a_2}\ln|x-m_1| + t_2 \ln|x-m_2| + C; C \in \mathcal{R} \tag{10}$$

Ordinary differential equations [2] can be considered as the second important topic in courses of mathematical analysis. Homogenous linear ordinary differential equations (LDE) of higher orders with constant coefficients can be written by equations in (11) [2]. This type of LDE can be solved using characteristic equation (12) [2], in which previous homogenous LDE is rewritten into a form of an algebraic equation. Degrees in polynomials of this characteristic equation are related to orders of corresponding derivations in original homogenous LDE (11).

$$\left.\begin{array}{c} v_n y^{(n)} + v_{n-1} y^{(n-1)} + \cdots + v_1 y' + v_0 = 0; \\ y = y(x); \boldsymbol{v} = [v_n \cdots v_0]^T; \boldsymbol{v} \in \mathcal{R}^{n,1} \end{array}\right\} \tag{11}$$

$$v_n \sigma^n + v_{n-1}\sigma^{n-1} + \cdots + v_1 \sigma + v_0 = 0 \tag{12}$$

Corresponding to complex roots (13) of characteristic equation (12), general solution of a homogenous LDE can be determined using rules: (14) for different roots, (15) for a multiplied root and (16) for complex roots. These rules can be combined appropriate together. Roots of equation (12) can be suitable fast solved using rule (17), which can be applied in Octave.

$$\boldsymbol{\sigma} = [\sigma_1 \cdots \sigma_n]^T; \boldsymbol{\sigma} \in C^{n,1} \tag{13}$$

$$\left.\begin{array}{c} \sigma_1 \neq \cdots \neq \sigma_n : y = C_1 e^{\sigma_1 x} + C_2 e^{\sigma_2 x} + \cdots + C_n e^{\sigma_n x}; \\ \boldsymbol{c} = [C_1 \cdots C_n]^T; \boldsymbol{c} \in \mathcal{R}^{n,1} \end{array}\right\} \tag{14}$$

$$\left.\begin{array}{c} \sigma_1 = \cdots = \sigma_n = \omega : y = C_1 e^{\omega x} + C_2 x e^{\omega x} + \cdots + x^{n-1} C_n e^{\omega x}; \\ \boldsymbol{c} = [C_1 \cdots C_n]^T; \boldsymbol{c} \in \mathcal{R}^{n,1} \end{array}\right\} \tag{15}$$

$$\left.\begin{array}{c} \sigma_{1,2} = \alpha \pm \beta\, i : y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x); \\ \boldsymbol{c} = [C_1\ C_2]^T; \boldsymbol{c} \in \mathcal{R}^{2,1} \end{array}\right\} \tag{16}$$

$$\boldsymbol{\sigma} = [\sigma_1 \cdots \sigma_n]^T \equiv \texttt{Roots}([v_n \cdots v_1\ v_0]) \tag{17}$$

Results

For purposes of a course focused on mathematical analysis in author's foreign-language education, following problems (18) & (24)-(26) were defined for a foreign student in frame of a final exam. Correct solutions were achieved by the student, as can be seen in procedures

(19)-(23) for problem (18) and in (24)-(26) for the second category of problems. Software Octave was suitable utilized for finding roots (19) & (24)-(26) and for solving set of linear equations (22) with advantageous application in Octave in favor of an improvement of student's knowledge.

The first part of an exam: "Solving integral using the method of undetermined coefficients"

$$\int \frac{1}{2x^2 + 10x + 12} dx \tag{18}$$

$$\left.\begin{array}{c} \boldsymbol{m} = [m_1 \ m_2]^T \equiv \texttt{Roots}([2 \ 10 \ 12]); \\ \boldsymbol{m} = [-3 - 2]^T \\ 2x^2 + 10x + 12 = 2(x+3)(x+2) \end{array}\right\} \tag{19}$$

$$\int \frac{t_1}{2(x+3)} dx + \int \frac{t_2}{x+2} dx; \boldsymbol{t} = [t_1 \ t_2]^T \tag{20}$$

$$\left.\begin{array}{c} \dfrac{1}{2(x+3)(x+2)} = \dfrac{t_1}{2(x+3)} + \dfrac{t_2}{x+2} \\ 1 = t_1(x+2) + 2t_2(x+3) \\ \underline{0}x + \underline{\underline{1}} = x(\underline{t_1 + 2t_2}) + \underline{\underline{2t_1 + 6t_2}} \end{array}\right\} \tag{21}$$

$$\left.\begin{array}{c} \boldsymbol{t} = \left[t_1 \ t_2\right]^T \equiv \texttt{inv}\,(\boldsymbol{D}) * \boldsymbol{\delta}; \\ \boldsymbol{Dt} = \boldsymbol{\delta}; \boldsymbol{D} \in \mathcal{R}^{2,2}; \boldsymbol{t} \in \mathcal{R}^{2,1} \ \boldsymbol{\delta} \in \mathcal{R}^{2,1} \\ \boldsymbol{D} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}; \boldsymbol{\delta} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \boldsymbol{t} = \boldsymbol{D}^{-1}\boldsymbol{\delta} \\ \boldsymbol{t} = \left[t_1 \ t_2\right]^T = \begin{bmatrix} -1 & \dfrac{1}{2} \end{bmatrix}^T \end{array}\right\} \tag{22}$$

$$\left.\begin{array}{c} \displaystyle\int \frac{-1}{2(x+3)} dx + \int \frac{\frac{1}{2}}{x+2} dx = -\frac{1}{2}\int \frac{1}{x+3} dx + \frac{1}{2}\int \frac{1}{x+2} dx = \\ \begin{vmatrix} x+3 = \mu_1 \\ dx = d\mu_1 \end{vmatrix}; \begin{vmatrix} x+2 = \mu_2 \\ dx = d\mu_2 \end{vmatrix} \\ \displaystyle = -\frac{1}{2}\int \frac{1}{\mu_1} d\mu_1 + \frac{1}{2}\int \frac{1}{\mu_2} d\mu_2 = \\ \displaystyle = -\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x+2| + C; C \in \mathcal{R} \end{array}\right\} \tag{23}$$

The second part of an exam: "Solving type examples of homogenous ordinary linear differential equations with constant coefficients":

$$\left.\begin{array}{c} y''' + 9y'' + 26y' + 24 = 0; \\ \sigma^3 + 9\sigma^2 + 26\sigma + 24 = 0; \\ \boldsymbol{\sigma} = [\sigma_1\ \ \sigma_2\ \ \sigma_3]^T \equiv \texttt{Roots}([1\ 9\ 26\ 24]) \\ \boldsymbol{\sigma} = [2\ \ 3\ \ 4]^T; \sigma_1 \neq \sigma_2 \neq \sigma_3 : \\ y = C_1 e^{2x} + C_2 e^{3x} + C_3 e^{4x}; \\ \boldsymbol{c} = \left[C_1\ \ C_2\ \ C_3\right]^T; \boldsymbol{c} \in \mathcal{R}^{3,1} \end{array}\right\} \tag{24}$$

$$\left.\begin{array}{c} y''' + 3y'' + 3y' + 1 = 0; \\ \sigma^3 + 3\sigma^2 + 3\sigma + 1 = 0; \\ \boldsymbol{\sigma} = [\sigma_1\ \ \sigma_2\ \ \sigma_3]^T \equiv \texttt{Roots}([1\ 3\ 3\ 1]) \\ \boldsymbol{\sigma} = [1\ 1\ 1]^T; \sigma_1 = \sigma_2 = \sigma_3 : \\ y = C_1 e^{x} + C_2 x e^{x} + C_3 x^2 e^{x}; \\ \boldsymbol{c} = \left[C_1\ \ C_2\ \ C_3\right]^T; \boldsymbol{c} \in \mathcal{R}^{3,1} \end{array}\right\} \tag{25}$$

$$\left.\begin{array}{c} y'' + y' + 1 = 0; \\ \sigma^2 + \sigma + 1 = 0; \\ \boldsymbol{\sigma} = [\sigma_1\ \ \sigma_2]^T \equiv \texttt{Roots}([1\ 1\ 1]) \\ \boldsymbol{\sigma} = [(-\frac{1}{2} - \frac{\sqrt{3}}{2}i)\ \ (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)]^T; \sigma_{1,2} = \alpha \pm \beta\, i : \\ y = e^{-\frac{x}{2}}(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x); \\ \boldsymbol{c} = \left[C_1\ C_2\right]^T; \boldsymbol{c} \in \mathcal{R}^{2,1} \end{array}\right\} \tag{26}$$

Conclusion

In frame of a particular preparation of future mathematics teachers, a practical utilization of described algebraic routines was proposed in favor of course of mathematical analysis. Presented approach implemented finding roots and solving set of linear equations in software Octave. If these algebraic routine procedures are supported by this free-available software, then following advanced parts of mathematical topics can be explained more effectively by a teacher. An automation of common algebraic operations can be appropriate in more difficult topics in mathematics, which are usually integrated in beginnings of study plans at some specialized foreign faculties of education; especially in Switzerland, in Estonia or in Turkey. The presented application could be also useful in favor of diploma theses of incoming foreign students in their home institutions.

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